Stephen Guerin <redfishgroupllc@gmail.com>

Diffracton: minding the gap

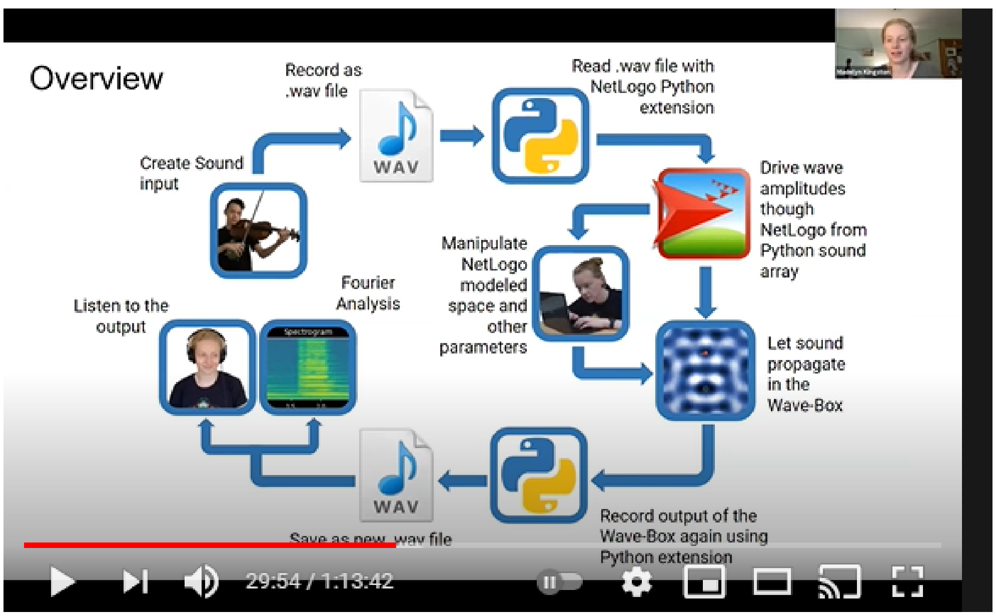
Stephen Guerin <stephen.guerin@simtable.com> Sat, Aug 14, 2021 at 10:17 AM
To: "Wedtech@Redfish. Com" <wedtech@redfish.com>, The Friday Morning Applied Complexity Coffee Group <friam@redfish.com>
Cc: Ed Angel <angel@cs.unm.edu>, Steve Smith <sas@lava3d.com>, Bruce Sherwood <bruce.sherwood@gmail.com>, Ruth Chabay <rwchabay@gmail.com>, Uri Wilensky <uri.wilensky@gmail.com>

At yesterday's Virtual Friam I asked a question on diffraction and said I would send more background.

The gist of my question is:

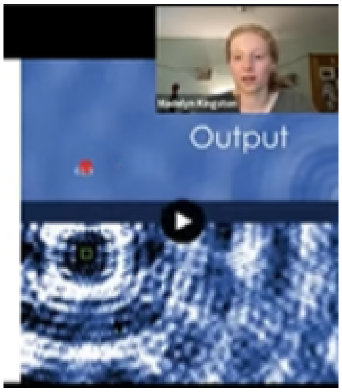
Even though I completely understand the micro-level rules that generate diffraction in the wave model described below, I still don't have an intuition **how the gaps in an obstacle have the emergent effect of diffracting waves when wavelengths >= gap width. Can anyone help?**

Background:
The question arose from my mentoring NM School for the Arts high school students in the NM Supercomputing Challenge where the students simulated spatial acoustics by appropriating Saint-Venant equations used for shallow water waves to instead model acoustic pressure waves. We wrote a Netogo agent-based model with Python extension for reading / writing the sound files and simulating spatial acoustics.



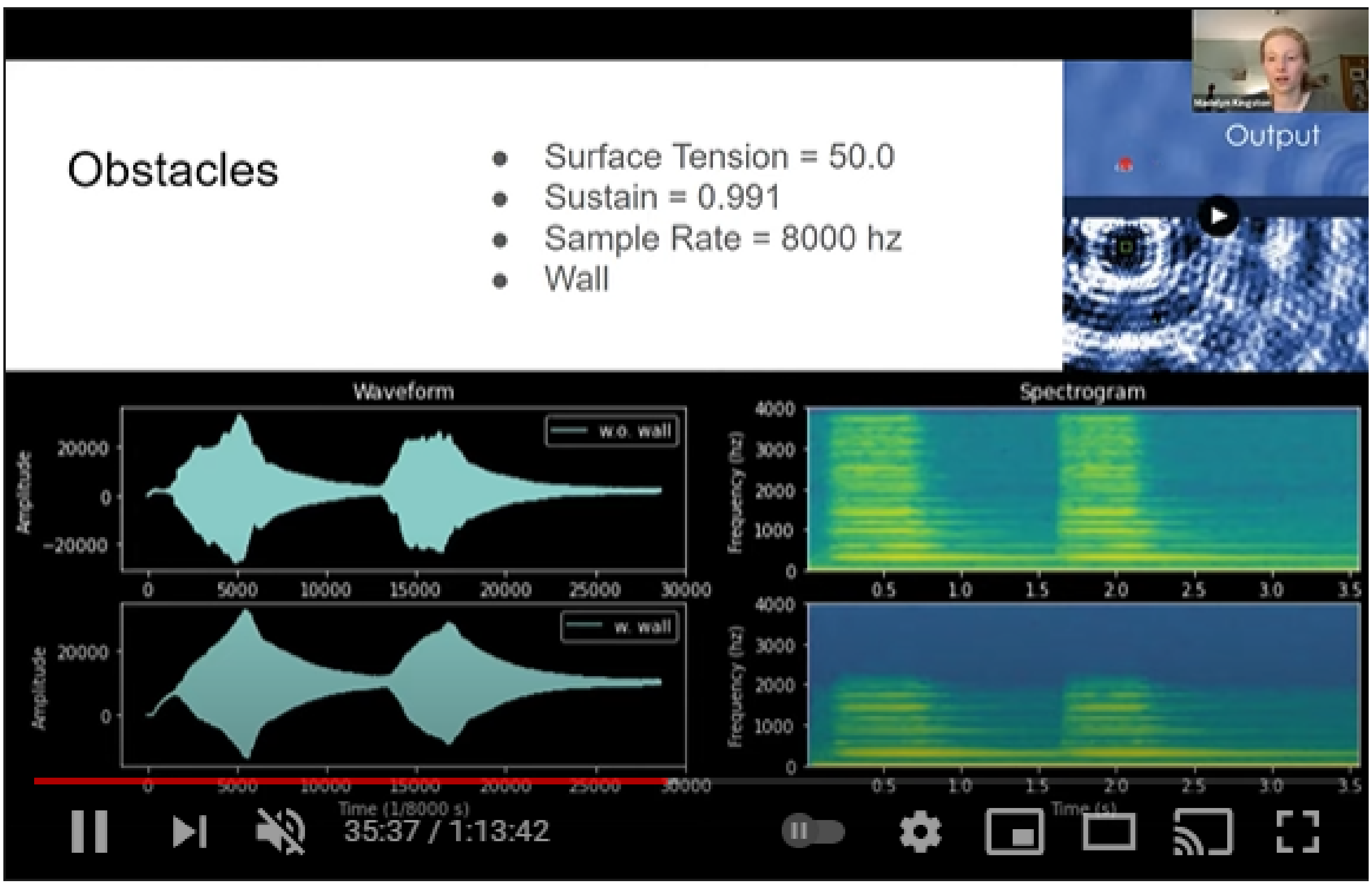
The students explored the effects of different room configurations on acoustics.

One configuration of interest was a wall gap illustrated below in the top right under Madelyn's video below. The wall gap is hard to see on right side.



They simulated microphones in Netlogo by recording amplitudes at a patch (red dot below in top-right visualization of room) and simulated speakers (hard-to-see blue dot below red dot on other side of wall) by driving amplitudes at a patch from the time series of amplitudes in .wav files (recordings of a singer and viola performance). They could hear, and through Fourier analysis, see the gap acting as a low-pass filter on the acoustic signal. ie, only the low frequencies were "bending" around the wall to reach the microphone.

You can see and listen to this effect and the spectrogram visualization at time 33:11 in their presentation.

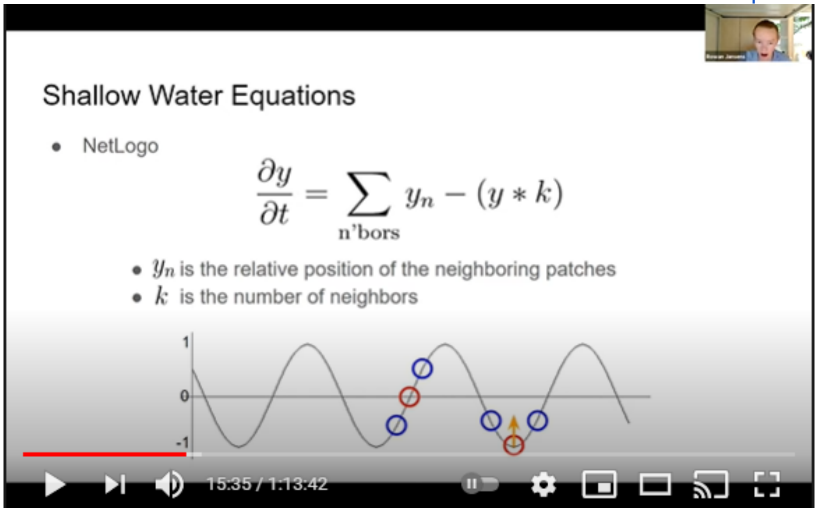


It took me a few weeks after their presentation in the NM Supercomputing Challenge - they got second place - to connect the low pass filter behavior to the concept of diffraction. Had this been a light model and I saw the rainbow effects I would have clued in much faster. Their presentation was a month after finals and they added this epilogue in the presentation above to identify the effect as diffraction.

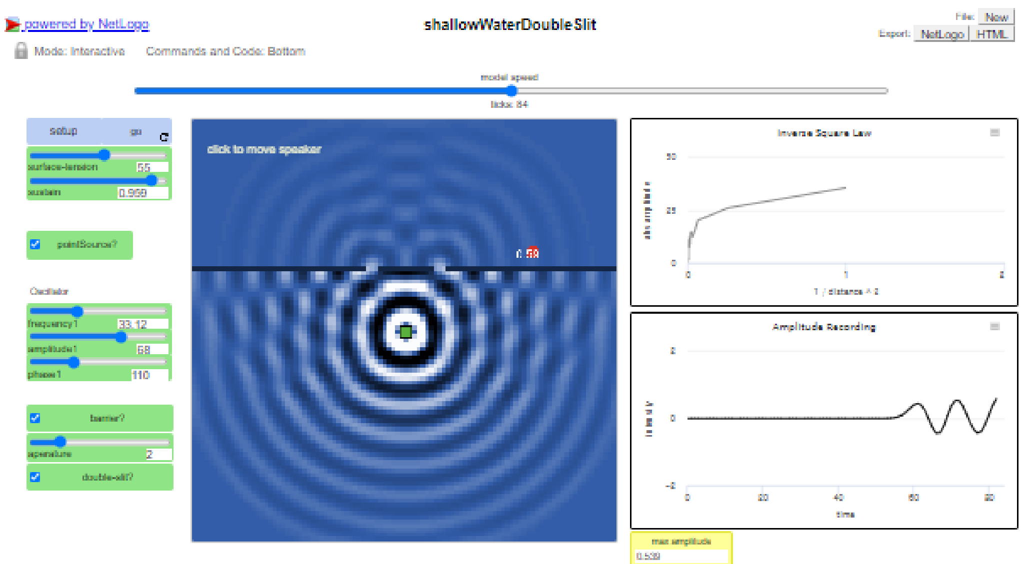
Their presentation included this physical wavepool video demonstration which was helpful to me to begin to understand the diffraction relationship with frequency and gap width.

Note: my question is not about "describing" the behavior with macroscopic equations or geometric models but fundamentally how does the gap become a point source ala Huygens Principle at the micro-level of the patches interacting with the emergent waves. To help with the distinction, I consider this interactive model a great macroscopic description of the phenomenon that nicely illustrates the relationship of frequency and gap width but doesn't help me interpret the micro-level interactions giving rise to the diffraction effect in our simple shallow-water model.

The students describe the details of the shallow water model at this point in their presentation:



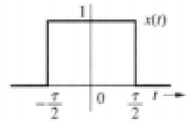
Here is my simplified Netlogo wave model of the same shallow water equations without the acoustics. It's set up to explore double slit but you can change it to single slit and mess with frequency and gap and watch the wave propagations, diffractions and interference patterns
https://anysurface.com/sguerin/models/shallowWaterDoubleSlit.html



As a related aside, with some follow-up discussions with Ed Angel and Steve Smith I am also trying to understand how the gap might be considered a sampling function on the signal. My intuition is that the diffraction of the wave creates a spreader Sinc function and the gap is Rect function which are Fourier duals. In some way, i see Nyquist-Shannon Sampling Theorem related to the gap. Note that diffraction creates a spreader function on the back wall in single gap experiments and the gap may be considered a Rect pulse when smaller than the wavelength.

Fourier Transform of $x(t) = \text{rect}(t/\tau)$

- Evaluation: $$X(\omega) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{\tau}\right) e^{-j\omega t}\, dt$$



- Since rect(t/τ) = 1 for -τ/2 < t < τ/2 and 0 otherwise

$$X(\omega) = \int_{-\tau/2}^{\tau/2} e^{-j\omega t}\, dt = -\frac{1}{j\omega}\left(e^{-j\omega\tau/2} - e^{j\omega\tau/2}\right) = \frac{2\sin\left(\frac{\omega\tau}{2}\right)}{\omega} = \tau\frac{\sin\left(\frac{\omega\tau}{2}\right)}{\left(\frac{\omega\tau}{2}\right)} = \tau\,\text{sinc}\left(\frac{\omega\tau}{2}\right)$$

$$\text{rect}\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau\ \text{sinc}\left(\frac{\omega\tau}{2}\right)$$

Bandwidth ≈ 2π/τ

L7.2 p689

Stephen.Guerin@Simtable.com
CEO, Simtable http://www.simtable.com
1600 Lena St #D1, Santa Fe, NM 87505
office: (505)995-0206 mobile: (505)577-5828
twitter: @simtable
zoom.simtable.com